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Energy Loss in Vehicle Collisions

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ABSTRACT

Reconstruction methods typically are based upon impact velocity changes computed by one of two approaches. These are damage based or crush measurement techniques and impulse and momentum equation solutions. Crush measurement techniques have an analytical foundation based to a large extent on point mass collision theory, limited primarily to collisions of vehicles with a common final velocity at the contact surface. Impulse and momentum methods can treat a full, 2-dimensional collision with arbitrary restitution and friction coefficients. As such their analytical foundation is much broader than damage based or crush measurement methods. The energy loss relationship and the tangential correction factor form an important part of the crush measurement methods. These two relationships are derived in a more general fashion than has been available.

These two approaches are compared in this paper. The comparison focuses on the ability to accurately calculate energy loss. Data from some of the RICSAC experimental collisions is analyzed using the crush measurement model and the impulse and momentum model. The latter gives consistently low estimates of energy loss with accuracies ranging from -33% to 0%. The crush measurement model gives energy loss estimates which range from -55% to 109% of the experimental values. Though the crush measurement technique accuracy depends directly upon the ability to estimate of energy loss, the method appears to be less accurate in this respect than the impulse and momentum method.

COMPUTATIONAL APPROACHES CURRENTLY USED FOR ACCIDENT RECONSTRUCTION fall into two categories. One depends directly upon data in the form of measurements of the vehicles' deformed surfaces and corresponding vehicle damage databases to estimate energy loss [2-7]. Velocity changes are calculated using this energy loss with simplified equations from impulse and momentum theory. A spinout model (post impact dynamics) typically is also used to relate vehicle rest positions to impact velocities. In this paper, this approach will be referred to as the C/M (Crush Measurement) technique. The most commonly referenced implementation of this method is the damage-only portion of CRASH3 [11]. The other approach currently used relies almost exclusively upon the principles of impulse and momentum for analysis of the impact phase and here is referred to as the I/M method. This approach [8-10] relies on coefficients such as the classical coefficient of restitution, coefficient of friction (actually an equivalent coefficient of friction) and possibly others such as a moment coefficient [13] and a tangential velocity ratio [9]. Generally, post impact dynamics is handled by an independent dynamic model. Each of the two approaches has advantages and disadvantages when compared to the other. C/M methods can handle some collisions which I/M methods cannot, such as head-on collisions with little or no post impact motion. I/M methods can handle sideswipe collisions for which C/M methods as currently packaged are not appropriate. The C/M methods have a larger damage database compared to the information available for selecting I/M coefficients.

Probably one of the major advantages of the I/M methods is a much firmer analytical foundation; fewer simplifying assumptions have been made in developing model equations. A full, planar model of the mechanics of an impact of two rigid bodies is available in the form of six linear algebraic equations [10]. Several other equally valid but less general models exist of varying complexity [13]. Another advantage of the I/M methods is that crush measurements do not have to be made. Some of the problems with the C/M models have been discussed in a well documented critique [18]. Additional questions are raised in this paper about some of the

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fundamental aspects of crush energy and the accuracy of the estimates of energy loss by the C/M method.

The next section of this paper contains a summary of the main steps of each method as used to carry out an accident reconstruction. Mention of investigative tasks common to both are omitted, such as site measurements and vehicle data selection. A following section then focuses specifically on the analytical relationship between velocity changes (ΔV's) and crush energy, upon which C/M methods are based. A basic assumption of the C/M method is that the measured deformation and calculated energy loss corresponds only to crush normal to the surface [11]. Tangential effects are to be "added in" or obtained with a correction term. It is shown that the relationship between normal, or compressive crush and tangential effects (such as friction and shear) has never been thoroughly formulated. A more exact relationship between ΔV and energy loss is presented. Finally, staged collision data is reviewed using both models. A question arises as to whether or not the C/M approach attains a desirable accuracy for reconstruction purposes.

APPROACHES TO ACCIDENT RECONSTRUCTION

A primary goal of most accident reconstructions is to determine quantitatively the initial speeds and/or directions of one or more vehicles prior to a collision. Various sources of information from the vehicles and accident site are used to supply accident specific data used for calculations. The major steps of the two approaches mentioned above, are outlined and discussed here. For a more general discussion of accident reconstruction, see [1].

Crush Energy Methods

Though variations among investigators and between classes of collisions often occur, the basic reconstruction procedure using C/M methods contains the following steps (not necessarily in this order):

1. Measure the crush deformation of both vehicles
2. Transform the crush measurements into energy losses with the use of experimental damage databases
3. Choose a principle direction of force (PDOF)
4. Calculate velocity change (ΔV) of each vehicle using energy loss
5. Independently determine velocities at vehicle separation just following impact
6. Using results from 3, 4 and 5, calculate vehicle velocities just prior to impact

A complete description of crush energy, its modelling and measurement is given in [12] and the reader is referred there for more information. Commercially available computerized versions of C/M methods are available [4,7,11]. A few comments follow on some of the above items in order to bring out some of the differences in approaches.

Databases used with crush measurement methods contain experimental plastic structural deformation data fit to linear curves using regression methods, as described in [12]. Ordinarily, the use of experimental data presents a potential for good accuracy and sensitivity. But some question exists as to whether or not this potential has been realized [18]. Damaged surfaces are topologically complicated. Although sophisticated procedures have been reported, the actual number of measurements is kept small for practical reasons. The number and type of measurements are also tied to the algorithm and database used to calculate energy loss for the given vehicle. Crush-energy data may or may not be available for the specific vehicle. Most data which is available is statistically averaged over numerous vehicles in a class. For these and other reasons, large errors can occur [18].

Once the crush energy is known, analytical relationships exist [11] to calculate ΔV_1 and ΔV_2. These are the magnitudes of the velocity change vectors. These equations are a major topic of this paper and are discussed in detail later. Directional information for the velocity changes is supplied by item 3, the choice of a PDOF. The PDOF seems to be misnamed. As used, it actually is the line of action of the resultant impulse of the forces developed during the impact, not the forces themselves. Forces change thoughout the impact whereas the impulse is more akin to an average force. The value of the PDOF used in a reconstruction by the C/M method is based heavily on the judgement of the analyst and can be critical to the accuracy of a reconstruction [18]. Combining the PDOF with the ΔV_1 and ΔV_2 then leads to the last step in the reconstruction process. If the final impact velocities (those leading to spinout, or post impact motion) can be estimated, then the vector ΔV's permit calculation of the initial velocities of one or both vehicles. Final impact velocities are usually determined from post accident scene data and analytical methods of vehicle dynamics. CRASH3 and other implementations have algorithms built in for making post impact velocity estimates.

Impulse/Momentum Methods

The details of these methods will not be discussed here. The reader is referred to a review [13]. Rather, a summarized description follows on how I/M methods are used in an accident reconstruction setting.

A typical procedure is the following:

1. Locate a normal and tangential coordinate system on the vehicles' crushed surfaces which corresponds to their relative orientation while in contact
2. Determine a coefficient of restitution and impulse ratio.
3. Determine velocities at vehicle separation just following impact.
4. Calculate the preimpact velocities using the equations of impulse and momentum and the information from items 1, 2 and 3.

The choice of the normal and tangential (n,t) coordinate system requires judgement of the analyst since the common surface of the vehicles changes throughtout contact. In some cases the choice can be difficult; frequently an undeformed surface can be used. The next step, choice of coefficients e and μ is related to the (n,t) coordinates. The coefficient of restitution, e, is a measure of the relative rebound speed of the two vehicles at separation in the normal, or n, direction. The coefficient μ (frequently called a friction coefficient) is the ratio of the tangential to normal impulses generated during the impact [10,13]. Both quantities e and μ are directly tied to the collision's energy loss. This is discussed later in more detail. Just as the crush energy in C/M methods must correspond to the specific vehicles and collision, e and μ must also. The data base for their selection is not very extensive, primarily restricted to the RICSAC collisions [14]. However, the situation is not bleak since μ possesses a maximum, or critical, value, which is reached in many (if not most) typical collisions. (Again, more later). Step 3 for I/M methods is basically the same as Step 5 for C/M methods. Step 1 and the selection of μ combine to determine the PDOF in the I/M approach. At Step 4 the I/M methods revert to a solution (usually computerized) of Newton's laws. This provides the preimpact velocities which correspond to the post impact, or separation, velocities from step 3. Sequential or iterative solutions are frequently necessary, but methods exist to eliminate the need for iterative solutions [15].

It is frequently mentioned that the ΔV's can be calculated by C/M methods even if vehicle separation velocities are not available. The equation which is used is actually based upon the I/M equations, which is now demonstrated.

ENERGY LOSS

According to Newton's laws, when two masses collide in the absence of external forces, momentum is always conserved; energy may or may not be. In this section of the paper, the energy loss is examined. The accuracy of the C/M methods is directly dependent upon the relationship between ΔV and the energy loss which of course must follow Newton's laws. The following analysis is highly summarized since the basics have been presented elsewhere [10,13]. The development of the equations is similar to that of others [12,16] whereby the analysis first is done for a collision of point masses and then later refined to include rigid body rotational effects.

The collision problem of 2 point masses has 4 unknown final velocity components V_{1n}, V_{1t}, and V_{2t}* and 4 known initial velocity components v_{1n}, v_{1t}, v_{2n}, v_{2t}. Four equations are available from the following concepts:

1. Conservation of momentum in the direction of the normal axis
2. Conservation of momentum in the direction of the tangential axis
3. restitution normal to the impact surface
4. "friction" in the tangential direction

It is important to note that the last equation is expressed as

$$P_t = \mu P_n \qquad (1)$$

where P_t is the tangential impulse, P_n is the normal impulse and μ is a constant equal to their ratio. This constant is not strictly a coefficient of friction since the tangential forces generated by colliding vehicles do not fit the usual definitions of Coulomb friction. Consequently, μ should be thought of simply as an impulse ratio or an equivalent friction coefficient.

The solution of the 4 equations is easy to obtain and gives the final velocities in terms of the initial. As a result, the total loss in kinetic energy of the system, T_L, can be expressed in terms of the masses, initial velocities and coefficients. This is

$$T_L = \frac{1}{2} \bar{m} (v_{2n} - v_{1n})^2 (1 + e) [(1 - e) + 2\mu r - (1 + e) \mu^2] \qquad (2)$$

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* Throughout this paper, capital V's represent final velocities and small, or lower case, v's represent initial velocities.

where $\bar{m} = m_1 m_2/(m_1 + m_2)$ and

$$r = \frac{v_{2t}-v_{1t}}{v_{2n}-v_{1n}} \tag{3}$$

Define a relative angle of approach, Ψ, such that

$$\tan \Psi = \frac{1}{r} = \frac{v_{2n}-v_{1n}}{v_{2t}-v_{1t}} \tag{4}$$

With this definition, a direct (head on) collision has a value of $\Psi = 90°$ and Ψ is near 0° for a side swipe.

In [13] it was shown that μ can be either positive or negative but is bounded, that is, $|\mu| \leq |\mu_m|$ where μ_m is the impulse ratio just sufficient to cause relative tangential motion (sliding) to cease at separation and where

$$\mu_m = \frac{r}{1+e} \tag{5}$$

Equality of μ to μ_m is what others [12] refer to as a common, final tangential velocity. These concepts and definitions allow T_L to be written as

$$T_L = \frac{1}{2} \bar{m}(v_{2n}-v_{1n})^2 \{ (1 - e^2) + r^2 [2(\frac{\mu}{\mu_m})-(\frac{\mu}{\mu_m})^2] \} \tag{6}$$

It is easy to recognize that the term, $(1-e^2)$, is that which corresponds to crush energy in the normal direction as a consequence of the definition of e. The term involving μ/μ_m corresponds to energy loss caused by relative tangential motion (shear, friction, metal to metal interference, etc.) Keeping this equation in mind, the velocity change ΔV_1 of mass 1 is now found (ΔV_2 is found similarly). Since ΔV_1 is a vector

$$\Delta V_1^2 = (V_{1n}-v_{1n})^2 + (V_{1t}-v_{1t})^2$$
$$\Delta V_1^2 = (1 + \mu^2) [\frac{\bar{m}}{m_1}(1+e)(v_{2n}-v_{1n})]^2 \tag{7}$$

where the final velocities V_{1n} and V_{1t} have been eliminated using the solution of the impact equations. The parts of eq 6 and 7 containing $\bar{m}(v_{2n}-v_{1n})^2$ can be eliminated giving

$$\Delta V_1^2 = \frac{1}{m_1^2} \{ \frac{2\bar{m} (1 + \mu^2)(1 + e)^2 T_L}{(1-e^2) + r^2[2(\frac{\mu}{\mu_m})-(\frac{\mu}{\mu_m})^2]} \} \tag{8}$$

This rather unwieldy expression is the general relationship mentioned earlier which relates the total kinetic energy loss, T_L, to ΔV_1 for point mass collisions. Consider the special case of e=0 and μ=0 which then makes $T_L = CE_T$, where CE_T is called the total crush energy in [12]. With these coefficient values Eq 8 can then be put into the form

$$\Delta V_1^2 = \frac{m_2}{m_1} \frac{2 CE_T}{m_1 + m_2} \tag{9}$$

This is the equation typically used to relate crush energy, CE_T, and velocity change [11,12] as used by C/M methods. It frequently appears in a form corrected for rigid body rotation effects [11] which will be done later. Note that Eq 9 corresponds to a perfectly plastic, frictionless collision. It is apparent that an error in CE_T will result in an error in ΔV_1. But note that omission of the terms involving e and μ independently can contribute additional error. This will be illustrated in the next section with the introduction of experimental results.

The use of Eq 9 (corrected for rotational effects) with CE_T from measured crush dimensions would be expected to underestimate the ΔV's since energy loss due to tangential effects is not taken into account. This has been recognized and a correction factor is included in CRASH3 [5,11]. The suggested correction, CF, is

$$CF = 1 + \tan^2 \alpha \tag{10}$$

where α is an angle between an intervehicular force and its component normal to the crush surface [11]. It is an unstated assumption in [11] that energy can be transformed (or corrected) according to the rules for force components, hence the tangent in Eq 10. Reconsider Eq 6. Let T_m be the maximum amount of energy which theoretically can be lost in a 2 mass collision [13]. T_m is

$$T_m = \frac{1}{2} \bar{m} [(v_{2n}-v_{1n})^2 + (v_{2t}-v_{1t})^2] \tag{11}$$

Using this and the definition of the angle Ψ (see Eq 4), permits the energy loss to be written as

$$T_L = T_m (1-e^2) \{\sin^2\Psi + [2(\frac{\mu}{\mu})-(\frac{\mu}{\mu_m})^2] \cos^2\Psi\} \tag{12}$$

or

$$T_L = T_m(1-e^2) \sin^2\Psi [1 + \frac{2(\frac{\mu}{\mu_m})-(\frac{\mu}{\mu_m})^2}{1-e^2} \tan^2\beta] \tag{13}$$

where $\beta = 90-\Psi$. The quantity in the brackets of Eq 13 can be compared directly to the CRASH3 correction factor, Eq 10, since $T_m (1-e^2)\sin^2\Psi$ is the energy loss due to crush alone, normal to the surface. The quantity in the brackets, Eq 13, is a more general correction. Note that the factor CF appears to be identical to the bracketed quantity of Eq 13 for e=0 and $\mu=\mu_m$. This corresponds to the vehicles reaching a common velocity at separation which is often a valid assumption. But note that $\tan^2\alpha \neq \tan^2\beta$ since α is an angle between some undefined forces whereas β is determined (through Ψ and r) by the initial relative velocity components.

Recall that all of the above equations follow from point mass theory. Typical practice [11,12] is to correct the equation for ΔV, Eq 9, to include the effects of rotational inertia. This gives

$$CE_T = \frac{1}{2}\frac{m_1}{m_2}\frac{(\gamma_1 m_1 + \gamma_2 m_2)}{\gamma_1\gamma_2}\Delta V_1^2 \qquad (14)$$

Where $\gamma_i = k_i^2/(k_i^2 + h_i^2)$; k_i is the yaw radius of gyration of vehicle i and h_i is the perpendicular moment arm from the line of action of the impulse to the mass center. A corresponding equation exists in terms of ΔV_2 and is not presented here. Also, keep in mind that CE_T is the total crush energy, which is the sum of the individual crush energies for each vehicle.

At this point, several comments can be made. C/M methods have a crucial dependence upon the relationship between each vehicle's ΔV and the energy loss in a collision. Such an important relationship should be as broadly or generally applicable as possible. Yet the equation currently used (Eq 14) has been shown to correspond only to the special case of $e = 0$ (no elastic rebound). The crush energy correction factor, CF, should also be as general as possible, yet Eq 10 corresponds to the special case of $e = 0$ and $\mu = \mu_m$. Although $\mu=0$ and $\mu=\mu_m$ give the same value to the correction term in Eq 13, they do not give the same value for ΔV in Eq 8. So although CE_T is corrected, Eq 9 is still a very special case of Eq 8. In addition, the angle α used with the correction factor, is actually undefined. It appears that the angle between the normal to the crush surface and the PDOF is typically used. But this angle can be incorrect. The appropriate angle is β which is determined by the initial velocity components as defined earlier.

Equations 8 and 13 provide a broader analytical foundation and the potential for more accurate results for C/M methods. Unfortunately these equations cannot be adapted because C/M methods do not use the coefficients e and μ so their values are not available. The converse is not true however. Anyone using I/M methods can easily use the measured crush energy. Since measured crush energy is intended to correspond only to normal energy loss, then $\mu=0$ and Eq 6 gives

$$T_L(\mu=0) = CE_T = \frac{1}{2}\bar{m}\,(v_{2n} - v_{1n})^2(1 - e^2) \qquad (15)$$

This can be solved for e^2;

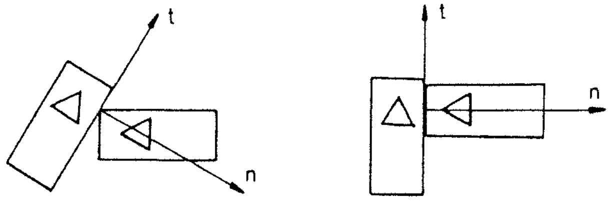
$$e^2 = 1 - \frac{2\,CE_T}{\bar{m}(v_{2n} - v_{1n})^2} \qquad (16)$$

and provides the value of e for an I/M solution which has the appropriate crush energy loss.

Also note that with the exception of Eq 14, all of the above theory is for point mass collisions; rigid body rotational effects have been neglected. This is because the corresponding theory which includes rotational effects provides long, complicated equations, some of which have never been published. It is well known that rotational effects can be significant. Methods based primarily on impulse and momentum, i.e., I/M methods, do not have this limitation since they typically use computerized solutions of equations which take rotational effects into account [9,10,13] and can be solved for any appropriate values of e and μ.

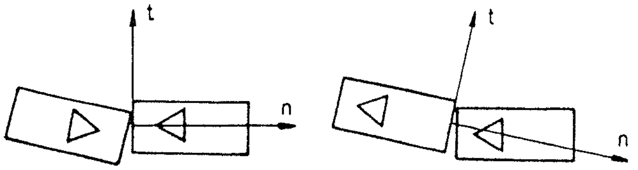
COMPARISONS WITH EXPERIMENTAL DATA

Data corresponding to the "RICSAC" staged collisions [17] are now reviewed in light of some of the above observations. Table 1 shows various quantities such as energy loss, impact coefficients, crush energy, etc. taken from various analyses [10,18] of 11 of the original RICSAC collisions. Fig 1 shows the preimpact vehicle orientations and illustrates the normal and tangential axes used in [10]. All data is grouped according to the RICSAC collision categories. The first two categories are those which can be expected to have relatively higher energy loss contributions due to tangential effects. In Table 1, The Initial System Energy and Experimental Energy Loss values are those obtained from the RICSAC reports [17] and are used here as a basis of comparison of the C/M and I/M methods. The Angle of Incidence, Ψ, and the corresponding Bracketed Quantity, Eq 13, correspond to the coordinate systems in Fig 1 and the earlier definitions of the angles Ψ and β. The next three rows of values in the table were provided in [18] and were calculated using C/M equations and algorithms from the data of [16]. The remaining rows of values correspond to data from [17] as analyzed in [10] using I/M equations. The results in Table 1 attributed to [18] and [10] are not attempts to "reconstruct" the RICSAC collisions. Instead they are intended to be analyses of experimental data using the respective models and provide an indication of how well the models fit the experimental data. In [10], initial velocities, measured final velocities, vehicle characteristics and collision parameters were used as input. The data was then fit to the impact model to yield the coefficients (impulse ratio and restitution). All energy loss terms thus correspond to the experimental data as it satifies the model's equations. That is, the energy losses are those of vehicles which satisfy Newton's laws of impulse and momentum and as closely as possible correspond to the experiments.



a. 60° FRONT TO SIDE

b. 90° FRONT TO SIDE



c. 10° FRONT TO FRONT

d. 10° FRONT TO REAR

Fig 1. Initial Vehicle Orientations for RICSAC Collisions

The same viewpoint can be taken concerning the results attributed to [18], but using the C/M method. For example, the PDOF'S used are those which were determined from the experimental data. These are used to compute the vector directions of the ΔV's as well as the angle α used in the tangential correction factor. The A, B and G coefficients [12] which provide the crush energy losses are also based on measurements of the RICSAC vehicles' damage. CRASH3 algorithms and databases for the appropriate vehicles were then used to calculate the crush energy loss values, CE_T. These results then, are those which satisfy the C/M damage-only model and simultaneously correspond to the experimental data.

The energy losses given in Table 1 correspond to the total value for both vehicles in a given collision. The Corrected Crush Energy values from the C/M model from Table 1 are illustrated in Fig 2. They are displayed as a fraction of the experimental values and grouped by collision category. The C/M model considerably overestimates energy loss in the 60° front to side category; in collision 6 by more than 100%. In other collision categories, all values but one are low. The deviations are rather large. From a statistical point of view, however, they seem to be systematic, rather than purely random. This suggests an error arising from the model which is dependent upon collision category, rather than simply experimental error.

The correction factors for tangential losses used by the C/M method (Ratio, Corrected/CE_T [18]) are considerably lower than the values calculated from Eq 13. This is true at least partly because Eq 13 arises from a point mass model which does not include off center effects. Even though the C/M model values are relatively low for the 60° front to rear category, the energy loss estimates are quite high for these collisions. This tends to indicate that the crush energy losses themselves, CE_T, are too large. Collisions 11 and 12 are almost direct central impacts and so the factors of 1 from Eq 13 should be accurate. The values indicated from the C/M approach are also 1, yet the energy losses are both considerably lower than the experimental values.

The corresponding data from the I/M model from Table 1 is illustrated in Fig 3. The trends here are quite different than those just examined. Firstly, the energy losses corresponding to the I/M method are either very close to the experimental values or are low. As from the C/M model, the I/M tangential energy loss values (Ratio, Impact/Normal [10]) are also lower than the point mass values for the first two collision categories, but they appear a bit more uniform within each catagory. The fact that the total energy losses are never higher than experimental values would tend to suggest that some energy loss mechanisms are neglected by the model. In fact, this is true since the assumptions corresponding to I/M theory presume that impulses due to forces other than the intervehicular forces are negligible. In addition, three dimensional effects (vertical motion) are neglected. Though little has been published on this latter topic, it is known that tire-to-ground frictional forces can be significant [9].

The first two collision categories are accompanied by rotational motion of the vehicles during contact. This indicates transverse skidding of at least some of the wheels. The last two catagories, 10° front to front and 10° front to rear should have little or no skidding effects since wheels were not locked. Table 2 includes approximate energy loss corrections for tire-ground friction based on point mass theory for the first 2 collision categories of the I/M results. With these additional energy losses included, the I/M values have been improved but still are low, ranging from -6 to -23%. How much of this remaining difference might be due to errors in the experimental values themselves is not known. The I/M values are significantly more accurate than the C/M values, however. They also have the feature of consistently underestimating the energy loss.

DISCUSSION AND CONCLUSIONS

Two major points are made in this paper concerning the generality and accuracy of damage-only or crush measurenment methods. The first is that two of the main analytical relationships, the one between ΔV and energy loss and the tangential correction factor, are based upon limited or special cases of impacts where $e = 0$ and $\mu = \mu_m$. The equations which form the basis of the C/M methods do not use explicitly the coefficients e and μ. Consequently, values other than $e = 0$ and $\mu = \mu_m$ cannot be used even when known. The second point is that when used with experimental or staged collision data, the results of the C/M model differ significantly both between and within collision catagories. Differences in energy loss between calculation and experiment range from -55% to +109%.

Reconstructions based upon impulse and momentum solutions do not seem to suffer the same restrictions and inaccuracies for energy loss calculations. I/M methods can be based upon a full, two-dimensional solution of Newton's Laws for any appropriate values of e and μ. Based on experimental data, the accuracy of energy loss values in comparison to C/M methods is not only much better, -33% to 0%, but the I/M model seems always to underestimate energy loss. When tire/ground friction effects are added, the error becomes -23% to 0%. In addition, the I/M methods can obtain solutions based upon crush energy measurements and databases, when desirable.

Based upon the above it would seem wise to combine the best features of both approaches. When crush energy losses are the only reliable data available for a reconstruction, then the generalized C/M equations can be used. When crush measurements either are not available or desirable, the I/M method should be used.

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TABLE 1. STAGED COLLISION DATA

RICSAC COLLISION COLLISION GEOMETRY	1	6	7	8	9	10	11	12	3	4	5
	60° FRONT TO SIDE			90° FRONT TO SIDE			10° FRONT TO FRONT		10° FRONT TO REAR		
INITIAL SYSTEM ENERGY [17]	100900	106300	178580	132820	107410	260340	109700	253300	74292	249170	242440
EXPERIMENTAL ENERGY LOSS [17]	**64877**	**58949**	**98756**	**71189**	**41352**	**100750**	**103231**	**229765**	**25482**	**128819**	**102793**
Ψ, ANGLE OF INCIDENCE, DEG	30	30	30	45	45	45	85	85	80	80	80
BRACKETED QUANTITY, EQ 13	4.0	4.0	4.0	2.0	2.2	2.2	1.0	1.0	1.0	1.0	1.0
CRUSH ENERGY, CE_T [18]	48392	45365	71995	34805	26732	32190	78127	122700	17634	118855	117020
CORRECTED CRUSH ENERGY [18]	**92013**	**123053**	**137651**	**55833**	**35920**	**45629**	**78908**	**124818**	**17646**	**120349**	**117035**
RATIO, CORRECTED/CE_T [18]	1.9	2.7	1.9	1.6	1.3	1.4	1.0	1.0	1.0	1.0	1.0
e, RESTITUTION COEFF [10]	.000	.000	.000	.079	.400	.419	.000	.100	.217	.045	.053
μ, IMPULSE RATIO [10]	.966	.824	.772	.413	.486	.590	.038	.031	-.065	-.050	-.090
μ/μ_m [10]	1	1	1	1	1	1	1	1	1	1	1
IMPACT ENERGY LOSS, % [10]	51.9	48.3	48.8	36.1	28.8	31.0	90.9	91.9	34.2	36.3	32.0
IMPACT ENERGY LOSS [10]	**52367**	**51343**	**87147**	**47948**	**30934**	**80705**	**99717**	**232801**	**25408**	**90449**	**77581**
NORMAL ENERGY LOSS [10]	19575	21047	37323	33072	16863	40092	100046	233561	25779	91196	78793
TANGENTIAL ENERGY LOSS [10]	32793	30296	49824	14876	14071	40613	-329	-760	-371	-747	-1212
RATIO, IMPACT/NORMAL [10]	2.7	2.4	2.3	1.4	1.8	2.0	1.0	1.0	1.0	1.0	1.0

NOTES:

1. ALL ENERGY VALUES EXPRESSED IN FT-LB
2. VALUES CITED TO [18] ARE CALCULATIONS USING EXPERIMENTAL DATA, ACTUAL PDOF FACTOR AND A, B & G COEFFICIENTS FROM [16]
3. VALUES CITED TO [10] ARE BASED UPON DATA FROM [17], FIT TO A PLANAR IMPACT MODEL USING THE METHOD OF LEAST SQUARES
4. THE BRACKETED QUANTITY, EQ 13, CORRESPONDS TO A SOLUTION OF THE IMPULSE MOMENTUM EQUATIONS FOR POINT MASSES

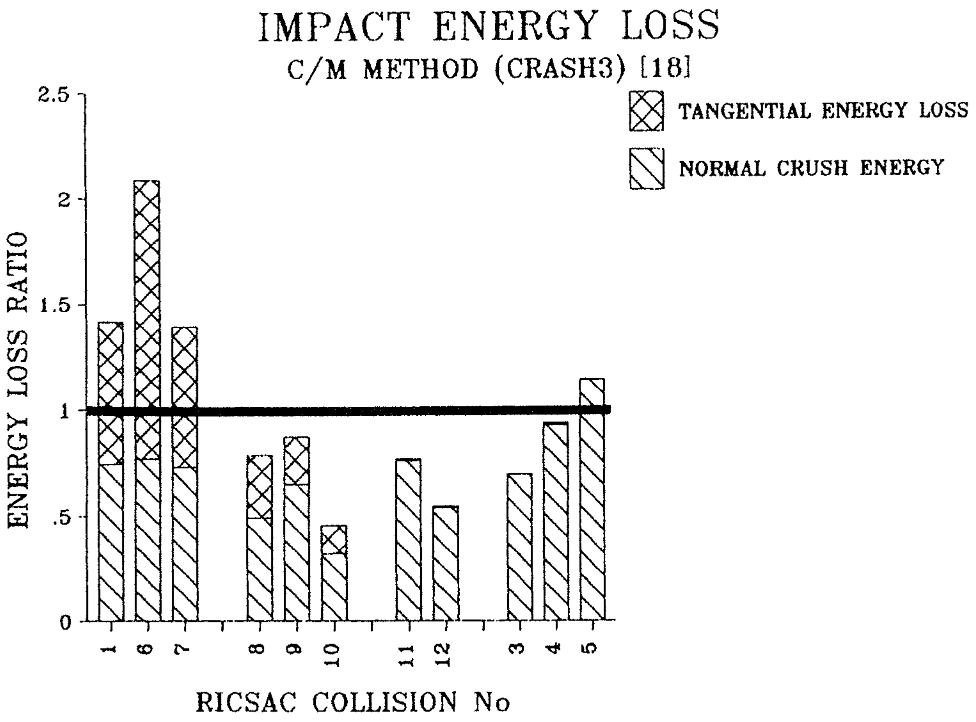

Fig 2. Ratio of Computed to Experimental Energy Loss, Crush Measurement Model

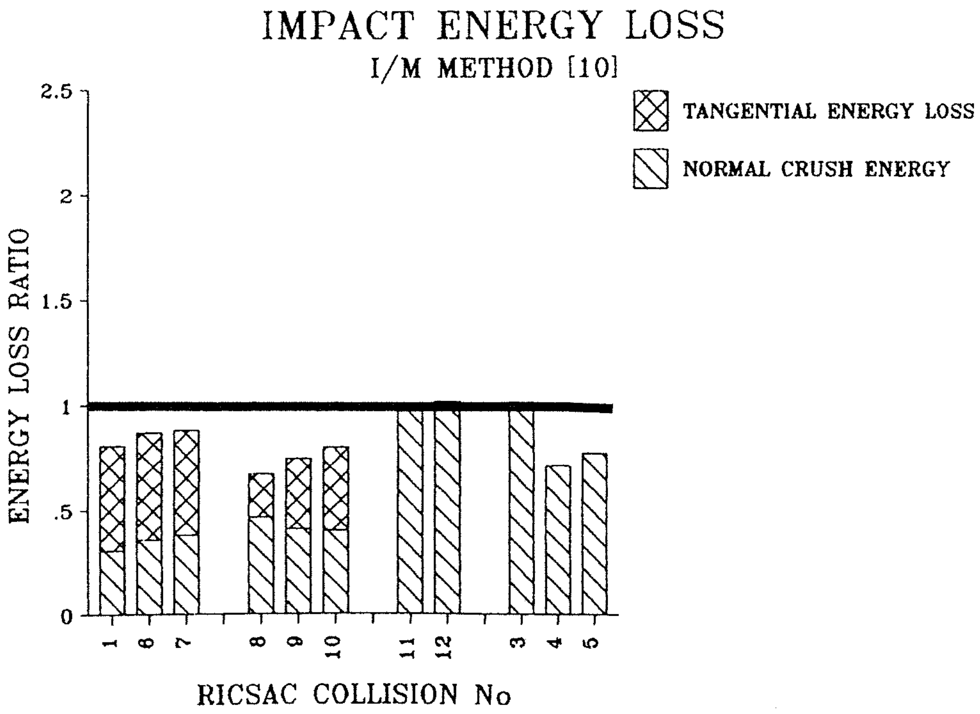

Fig 3. Ratio of Computed to Experimental Energy Loss, Impulse/Momentum Model

TABLE 2. ENERGY LOSS
INCLUDING TIRE-GROUND FRICTION
FROM I/M MODEL

RICSAC COLLISION	1	6	7	8	9	10
COLLISION GEOMETRY	60° FRONT TO SIDE			90° FRONT TO SIDE		
INITIAL SYSTEM ENERGY [17]	100900	106300	178580	132820	107410	260340
EXPERIMENTAL ENERGY LOSS [17]	**64877**	**58949**	**98756**	**71189**	**41352**	**100750**
IMPACT ENERGY LOSS [10]	52367	51343	87147	47948	30934	80705
TIRE-GROUND FRICTION LOSS	4382	4053	4953	6979	5688	9501
TOTAL ENERGY LOSS	**56749**	**55396**	**92100**	**54897**	**36622**	**90206**
DIFFERENCE (TOTAL/EXPM'L), %	-13	-6	-7	-23	-11	-10